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## **Estimating Exchange Rate Volatility- Comparative Study (Evidences from Sudan)**

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### **ABSTRACT**

This study aims to estimate the volatility of exchange rate through applying the GARCH model method which is appropriate in such cases. The study used symmetric and a symmetric model to the data of exchange of Sudan comparing two periods after and before separation of the country. The study found that the first period which is before separation is stable compare to the second period as indicated by GARCH findings. More over the second period which is after separation has leverage effect which means more risky to the investors than the first period. Finally the study attributed the relative stability of the first period to the massive production of oil in the southern region and consequently the return of hard currency from exportation of oil.

**Keywords:** Exchange Rate, GARCH symmetric model, GARCH Asymmetric Model

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### **INTRODUCTION**

The issue of exchange rate is hot one in current economic life because it considered as financial asset into contract. Each country have to manipulate its daily exchange rate in order to equilibrate the macro economic variables in accordance with policy targets. The system of exchange rates whether fixed or flexible significantly affected the inflow and outflow of capital, consequently, investment environment will establish. Furthermore the financial sector (money market) response to exchange rate state.

As far as economic policy effectiveness the system of exchange determines the power of fiscal, monetary, as well as mixed policy. So in order to tackle certain economic policy ought to be the count for the position of exchange rate, in effect, disturbances in exchange rates will worsen the targeted policies.

Interest rates determine by exchange rates state and consequently the prices levels adapt accordingly. Hence income redistribution readjusted in the sense of the above rates which play as appetizers to economic prone, where her exchange rate stands as leading steer in economization process.

Estimating and analyzing exchange volatility is crucial in realizing the current as well as future economic situations. So the true picture about economic situations help policy makers in tackling effective policy measures an so achieving macro as well as micro targets. Accordingly manipulating exchange rates data via strong econometrics technique can facilitate the matter of analysis , forecasting , as well as policies implementation.

### **VOLATILITY**

The three main purposes of forecasting volatility are for risk management, for asset allocation, and for taking bets on future volatility. A large part of risk management is measuring the potential future losses of a portfolio of assets, and in order to measure these potential losses, estimates must be made of future volatilities and correlations.

Financial markets data often exhibit volatility clustering, where time series show periods of high volatility and periods of low volatility. In fact, with economic and financial data, time-varying volatility is more common than constant volatility, and accurate modeling of time-varying volatility is of great importance in financial engineering. Increasingly however, econometricians are being asked

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to forecast and analyze the size of the errors of the model. In this case, the questions are about volatility, and the standard tools have become the ARCH/ GARCH models.

The basic version of the least squares model assumes that the expected value of all error terms, when squared, is the same at any given point. This assumption is called homoskedasticity, and it is this assumption that is the focus of ARCH/ GARCH models. Data in which the variances of the error terms are not equal, in which the error terms may reasonably be expected to be larger for some points or ranges of the data than for others, are said to suffer from heteroskedasticity. The standard warning is that in the presence of heteroskedasticity, the regression coefficients for an ordinary least squares regression are still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. Instead of considering this as a problem to be corrected, ARCH and GARCH models treat heteroskedasticity as a variance to be modeled. As a result, not only are the deficiencies of least squares corrected, but a prediction is computed for the variance of each error term. This prediction turns out often to be of interest, particularly in applications in finance.

### **CAUSES OF HETEROSKEDASTICITY**

Lamoureux and Lastrapes(1990). They mention that the conditional heteroskedasticity may be caused by a time dependence in the rate of information arrival to the market. They use the daily trading volume of stock markets as a proxy for such information arrival, and confirm its significance.

- Mizrach (1990). He associates ARCH models with the errors of the economic agents' learning processes. In this case, contemporaneous errors in expectations are linked with past errors in the same expectations, which is somewhat related with the old-fashioned “adaptable expectations hypothesis” in macroeconomics.
- Stock (1998). His interpretation may be summarized by the argument that “any economic variable, in general, evolves on an ‘operational’ time scale, while in practice it is measured on a ‘calendar’ time scale. And this inappropriate use of a calendar time scale may lead to volatility clustering since relative to the calendar time, the variable may evolve more quickly or slowly” (Bera and Higgins, 1990, p. 329; Diebold, 1986).

### **INFORMATION CHALLENGE**

The econometric challenge is to specify how the information is used to forecast the mean and variance of the return, conditional on the past information. While many specifications have been considered for the mean return and have been used in efforts to forecast future returns, virtually no methods were available for the variance before the introduction of ARCH models. The primary descriptive tool was the rolling standard deviation. This is the standard deviation calculated using a fixed number of the most recent observations. It is convenient to think of this formulation as the first ARCH model; it assumes that the variance of tomorrow's return is an equally weighted average of the squared residuals from the last 22 days. The assumption of equal weights seems unattractive, as one would think that the more recent events would be more relevant and therefore should have higher weights. Furthermore the assumption of zero weights for observations more than one month old is also unattractive.

### **ARCH MODEL**

The ARCH model proposed by Engle (1982) let these weights be parameters to be estimated. Thus, the model allowed the data to determine the best weights to use in forecasting the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual. Such an updating rule is a simple description of adaptive or learning behavior and can be thought of as Bayesian updating.

The ARCH models introduced by Engle (1982) which targeted at modeling and forecasting the error. The variance of the error considers as random conditional variable depending on its past observations. The mathematical structure of the model is as follow:-

ARCH(p,q) :

$$\varepsilon_t = \nu_t \sqrt{h_t} \dots\dots\dots (01)$$

$$h_t = c + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 \dots\dots\dots (02)$$

$$\nu_t \xrightarrow{iid} N(0,1)$$

Where;

$h_t$  is unconditional variance across time

$\nu_t$  random variable with identical independent distribution

N(0,1): Function of identical independent distribution

Relation (01) is result of heteroskedasticity hypothesis. The residual ( $\varepsilon_t$ ) is considered as multiplication of white noise ( $\nu_t$ ) in deviation of a random variable (from normal position)  $\sigma_t = \sqrt{h_t}$ , the last term is autoregressive for square of residual terms

ARCH test can be done by the following two hypotheses

$$H_0 = \alpha_0 = \alpha_1 \dots = \alpha_p = 0$$

$$H_1 = \exists \alpha_i \neq 0$$

After estimating ARCH model lagrange multiplier value is calculated by:-

$$LM_{cal} = N * R^2 \dots\dots\dots (3)$$

Where;

N is number of observations and  $R^2$  is coefficient of determination of ARCH model with p lags

### **GARCH MODEL**

A useful generalization of this model is the GARCH parameterization introduced by Bollerslev (1986). This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. It gives parsimonious models that are easy to estimate and, even in its simplest form, has proven surprisingly successful in predicting conditional variances. The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance.

This models introduced by Bollerslev (1986) according to this model the return(amenable for statistical manipulation) of a financial asset as :-

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \dots\dots\dots (03)$$

Where;

$R_t$  is return in period t which is a random variable

Ln is natural logarithm with 2.718 base

$S_t$  is the price of asset in period t

According to ARCH model the return is random variable depends on its standard deviation and white noise written as:-

$$R_t = \sqrt{h_t} \nu_t \dots\dots\dots (04)$$

$$\nu_t \xrightarrow{iid} N(0,1)$$

The GARCH model in this case has the following form:

$$h_t = \alpha + \sum_{j=1}^p \beta_j h_{t-j} + \sum_{k=1}^q \gamma_k R_{t-k}^2 \dots\dots\dots (05)$$

The GARCH(1,1) as special case written as:-

$$h_t = \alpha + \beta h_{t-1} + \gamma R_{t-1}^2 \dots\dots\dots (06)$$

### **MODELS COMPARISON**

Recent developments in financial econometrics suggest the use of nonlinear time series structures to model the attitude of investors toward risk and expected return. For example, Bera and Higgins (1993, p.315) remarked that “a major contribution of the ARCH literature is the finding that apparent changes in the volatility of economic time series may be predictable and result from a specific type of nonlinear dependence rather than exogenous structural changes in variables.” In the case of financial data, for example, large and small errors tend to occur in clusters, i.e., large returns are followed by more large returns, and small returns by more small returns.

This suggests that returns are serially correlated.

Linear Time Series shocks are assumed to be uncorrelated but not necessarily identically independent distributed (iid). Nonlinear Time Series: shocks are assumed to be iid, but there is a nonlinear function relating the observed time series. Linear Time Series shocks are assumed to be uncorrelated but not necessarily identically independent distributed (iid).

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ARCH models are simple and easy to handle, take care of clustered errors, take care of nonlinearities and take care of changes in the econometrician’s ability to forecast.

ARMA models are used to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant. In market characterized by volatility an ARMA model cannot capture this type of behavior because its conditional variance is constant. So we need better time series models if we want to model the none constant volatility.

GARCH time series models that are becoming widely used in econometrics and finance because they have randomly varying volatility. ARCH is an acronym meaning AutoRegressive Conditional heteroskedasticity. In ARCH models the conditional variance has a structure very similar to the structure of the conditional expectation in an AR model we look at GARCH (Generalized ARCH) models that model conditional variances much as the conditional expectation is modeled by an ARMA model. so that the conditional variance at time t is the weighted sum of past squared residuals and the weights decrease as you go further back in time.

In introductory statistics, it is often mentioned that independence implies zero correlation but not vice versa. A process, such as the GARCH processes, where the conditional mean is constant but the conditional variance is none constant is an example of an uncorrelated but dependent process. The dependence of the conditional variance on the past causes the process to be dependent. The independence of the conditional mean on the past is the reason that the process is uncorrelated. AR(1) process has a none constant conditional mean but a constant conditional variance, while an ARCH(1) process is just the opposite. If both the conditional mean and variance of the data depend on the past, then we can combine the two models. In fact, we can combine any model with any of the GARCH models as combination of an AR(1) model with an ARCH(1) model.

A deficiency of ARCH(q) models is that the conditional standard deviation process has high-frequency oscillations with high volatility coming in short bursts. GARCH models permit a wider range of behavior, in particular, more persistent volatility

### **ADAPTATION IN GARCH MODEL**

Researchers have long noticed that stock returns have “heavy-tailed” or outlier-prone” probability distributions, and we have seen this ourselves in earlier chapters. One reason for outliers may be that the conditional variance is not constant, and the outliers occur when the variance is large. In fact, GARCH processes exhibit heavy tails even if *residual* is Gaussian. Therefore, when we use GARCH

models, we can model both the conditional heteroskedasticity and the heavy-tailed distributions of financial markets data. Nonetheless, many financial time series have tails that are heavier than implied by a GARCH process with Gaussian *residual*. To handle such data, one can assume that, instead of being Gaussian white noise, *residual* is an i.i.d. white noise process with a heavy-tailed distribution.

The similarities seen in this chapter between GARCH and ARMA models are not a coincidence. If  $h_t$  is a GARCH process, then  $h_t^2$  is an ARMA process but with weak white noise, not i.i.d. white noise. The capability of the GARCH(1,1) model to fit the lag-1 autocorrelation and the subsequent rate of decay separately is important in practice. It appears to be the main reason that the GARCH(1,1) model fits so many financial time series.

### ASYMMETRIC MODEL

In some financial time series, large negative returns appear to increase volatility more than do positive returns of the same magnitude. This is called the *leverage effect*. Standard GARCH models, that is, the models given by (05), cannot model the leverage effect because they model  $\sigma^2$  as a function of past values of  $a_t^2$  |whether the past values of  $a_t$  are positive or negative is not taken into account. The problem here is that the square function  $x^2$  is symmetric in  $x$ . The solution is to replace the square function with a flexible class of nonnegative functions that include asymmetric functions.

The EGARCH introduced by Nelson (1991) The researcher found out that the function of conditional variance is exponential none-linear as opposite to Bollerslev model. The model of a symmetric conditional heteroskedasticity written as :

$$\log(h_t) = \omega + \beta_j \sum_{j=1}^p \log(h_{t-j}) + \alpha_k \sum_{k=1}^q \frac{|R_{t-k}|}{\sigma_{t-i}} + \gamma_k \sum_{k=1}^q \frac{R_{t-k}}{\sigma_{t-i}} \dots \dots \dots (07)$$

Where  $\gamma_k$  measures the leverage effect. This effect exists when  $\gamma_k$  is negative and significant.

### FORECAST ISSUE

Forecasting ARMA/GARCH processes is in one way similar to forecasting ARMA processes the forecasts are the same because a GARCH process is weak white noise. What differs between forecasting ARMA/GARCH and ARMA processes is the behavior of the prediction intervals. In times of high volatility, prediction intervals using a ARMA/GARCH model will widen to take into account the higher amount of uncertainty. Similarly, the prediction intervals will narrow in times of lower volatility. Prediction intervals using an ARMA model without conditional heteroskedasticity cannot adapt in this way. To illustrate, we will compare the prediction of a Gaussian white noise process and the prediction of a GARCH(1,1) process with Gaussian innovations. Both have an ARMA(0,0) model for the conditional mean so their forecasts are equal to the marginal mean, which will be called  $\mu$  For Gaussian white noise, the prediction limits are  $\mu \pm z_{\alpha} / 2^R$  where R is the marginal standard deviation. For a GARCH(1,1) process  $\{y_t\}$ , the prediction limits at time origin  $n$  for  $k$ -steps ahead forecasting are  $\mu \pm z_{\alpha} / 2^R \sqrt{n + k|n}$  where  $\sqrt{n + k|n}$  is the conditional standard deviation of  $Y_{n+k}$  given the information available at time  $n$ . As  $k$  increases,  $\sqrt{n + k|n}$  converges to R, so for long lead times the prediction intervals for the two models are similar. For shorter lead times, however, the prediction limits can be quite different. In case of none normal conditional returns better modeling the excess kurtosis that observe with asset prices, the assumption that the conditional returns are normally distributed can be relaxed. For example, we can assume returns follow a student's t-distribution or a Generalized Error Distribution (GED), both of which can have fat tails.

### THE DATA

The time scope of the study extends from 2000 to 2015, the study depends on monthly data to exchange rate . As matter of fact the available rates is official ones determine by central bank of Sudan according to a variety of criteria , then the final daily rate is weighted average . With the requirements of comparison the study divided the time path into two distinguished periods. The first period from January 2000 to December 2010 while the second period from January 2011 to Jun 2015. The first period characterized by political stability, stable and appreciated exchange rate, foreign investments as well as low rates of inflation and controllable payment balance, above all the tow regions of the country are united. The period characterized by separation of south region from north



one, serious difficulties in payment balance and consequently the exchange rate as result of excluding the north budget from oil resources. Also this period witnessed political instability in addition of large capital outflow from the country. Here the study seeds the test such incidences from the observed data of monthly exchange rates via GARCH models.

	2000_2010 (First period)	2011_2015 (Second period)
Mean	2391.678	5966.361
Std Dev	222.3710	1628.548
Skewness	-0.498901	-0.570719
Kurtosis	1.737581	1.669802
Jerque Bera	14.24120	6.912697
prob	0.0000	0.031545

The statistics of the two periods reveal that the mean and standard deviation of first period is less than the second period, in addition, the first period is less skewed than the second period. Moreover the coefficients of kurtosis of the two periods is less than three in affect the coefficient of the first period is lower compare to the second one. The overall distributions of two periods are none-normal as they show by Jerque Bera coefficients and they respective probabilities. From the different individuals statistics one can detect that the first period is less volatile than the second period

	Garch(1,1)GED parm fixed at (1.5) (First period)			Garch M (std Dev)-GED parm fixed at (1.5) (Second period)		
	coefficient	Z	Prob	coefficient	z	prob
C	0.0000018	5.302463	0.0000	-0.000275	-10.23763	0.0000
Arch	0.436194	3.491622	0.0005	-0.007351	-3.109472	0.0019
Garch	0.697557	15.64731	0.0000	1.091484	51.16201	0.0000
Akiake (CI)		-6.892002			-2.920590	
Arch test (LM)		0.856350	0.3565		0.034972	0.8524

From the above table which exhibits symmetric models of Garch (the results of decreasing and increasing prices were same to future volatility or no count to the leverage effect) we notice that the constant which refers to the long run variance is significant for the two periods, worthy the variance of the first period is less than the second one in absolute value which might indicates that the second period is more volatile than the first one. As far as the Arch & Garch coefficients , garch coefficient greater than arch one for the two periods which indicates the dependence on near information or near price periods compare to later periods , this significantly indicates persistent of volatility of the two periods which is consolidated by the significance of arch and garch coefficient , the former indicates the presence of arch incidence (heteroskedasticity ) while the second indicate conditional variance in return. More over the sum of the arch & garch coefficients for the two periods are greater than one which also indicate that the two periods were not stable or shocks decay quickly in the future . Finally Garch coefficient in second period is greater than the one in the first period which might indicate that the conditional variance in the second period is strongly persist than that one in the first period. The test of heteroskedasticity done by LM (Lagrange multiplier)the incidence had been removed by models in the two periods as were shown by the value of probabilities greater than 5%. The last results empower the model fitness beside the value of Akiake criterion information.

	Egarch -GED parm fixed at (1.5) (First period)			Egarch-student's t (Second period)		
	coefficient	z	Prob	coefficient	z	prob
C	-1.2939	-10.91005	0.0000	0.199875	28.48931	0.0000
(Abs Residual)Arch	0.647314	8.85047	0.0000	-0.437654	-310.3205	0.0000
(residual) Asymmmetric	0.099886	2.538421	0.01	-0.223806	-1.328148	0.1841
(log) Garch	0.903273	88.09596	0.0000	0.978219	192.8616	0.0000
Akiake (CI)		-6.798774			-4.050991	
Arch test (LM)		0.348420	0.5560		0.070813	0.7913

From the above table which exhibits asymmetric models of Garch which called leverage effect (the results of decreasing prices generates more future volatility than increasing price) we notice that the

constant which refers to the long run variance is significant for the two periods, worthily the variance of the first period is less than the second one which might indicates that the second period is more volatile than the first one. As far as the Arch & Garch coefficients , garch coefficient greater than arch one for the two periods which indicates the dependence on near information or near price periods compare to later periods , this significantly indicates persistent of volatility of the two periods which is consolidated by the significance of arch and Garch coefficient , the former indicates the presence of arch incidence (heteroskedasticity ) while the second indicates conditional variance in return. Garch coefficient in second period is greater than the one in the first period which might indicate that the conditional variance in the second period is strongly persist than that one in the first period. As far as asymmetry test the incidence is existed in the second model as the coefficient is negative but not significant while in the first period the incidence of leverage effect is not exist . The existence of leverage effect in the second period (unfortunately not significant) indicates that the second period is more risky than the first one. The test of heteroskedasticity done by LM (Lagrange Multiplier) revealed that the incidence had been removed by models in the two periods as were shown by the value of probabilities greater than 5%. The last results empower the model fitness beside the value of Akiake criterion information.

## **RESULTS**

- (1) The data of the second period is more skewed and exhibited kurtosis incidence as compare to the second one.
- (2) The standard deviation of the second period is greater than the one of the first period which indicates that second period data is more scattered.
- (3) Mean exchange in first period is smaller than that one in the second period which means that exchange in first period has small depreciation compare to the second.
- (4) Second period has relatively large long run variance as compare to the first period.
- (5) The two periods characterized by volatility as indicated by significant arch coefficients.
- (6) The conditional variance which affects the returns existed in the two periods as indicated by significant garch coefficients.
- (7) The models of the two periods confirmed the persistence of skocks in the future with fast decay rate.
- (8) The fitted models in different period had removed heteroskedasticity incidence as shown by insignificant Lagrange Multiplier values.
- (9) Second period experienced by Leverage effect but not significant while the first period did not.
- (10) Second period is more risky as outlined by leverage effect so it is more costly to investors compare to the first period because exchange rate is address of internal market.

## **CONCLUSION**

From the above mentioned results one can conclude that the first period is relatively more stable than the first one. Such stability which supported by figures can attributed to the following reasons:-

- (1) Stable or even appreciated exchange rates as result to huge oil production in south region.
- (2) Prevailing of pace as result to pace agreement between the two regions and hence political stability.
- (3) Improved balance of payment position as result to hard currencies provided by oil exportation.
- (4) Inflow of foreign investments as result to conducive investment environment.

The above reasons classified the Sudan economy as rental one or not diversified economy according to the dependency on oil exportation, so as separation between the two regions happened and majority of oil went beside south region , north economic problems exaggerated as shown by second periods figures.

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